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BPS Domain Walls in Models with Flat Directions

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Abstract

We consider BPS domain walls in four-dimensional $\mathcal{N} = 1$ supersymmetric models with continuous global symmetry. Since the BPS equation is covariant under a global transformation, the solutions of the BPS walls also have global symmetry. The moduli space of the supersymmetric vacua in such models has non-compact flat directions, and complex BPS walls interpolating between two disjoint flat directions can exist. We examine this possibility in two models with global $O(2)$ symmetry and construct the solutions of such BPS walls.

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1 Introduction

In recent years, there has been much investigation of domain walls which appear in many areas of physics. These domain walls interpolate between degenerate discrete minima of a scalar potential, with dependence on one spatial coordinate. They can occur naturally when a discrete symmetry is spontaneously broken.

Domain walls can also appear in supersymmetric field theories when the superpotential has more than two critical points corresponding to degenerate minima of the scalar potential. In particular, it has been found that domain walls in supersymmetric theories can saturate the Bogomol'nyi bound. [1] Such domain walls are called BPS domain walls and preserve half of the original supersymmetry. [2] The existence of BPS domain walls corresponds to the central extension of $\mathcal{N} = 1$ superalgebra, and the topological charge of the walls becomes the central charge Z of the superalgebra. [3]-[5] The BPS bound and supercharges are determined by this central charge Z .

BPS domain walls in supersymmetric theories have been extensively studied in models with degenerate isolated vacua. [6]-[9] Moreover, it has been found that such BPS domain walls can form a junction when three or more different isolated vacua occur in separate regions of space. The BPS state of the junction preserves 1/4 supersymmetry, and the BPS bound is determined by two kinds of central charges, Z and Y , appearing in the $\mathcal{N} = 1$ superalgebra. [10] There has been progress in the study of the general properties of such BPS junctions, [11]-[17] for example, the negative contribution of the charge Y to the junction mass [12] and the non-normalizability of zero modes on the BPS junction. [16] It has also been argued that BPS junctions can create a network [11] and that they can play a role in our world in higher-dimensional spacetime with a negative cosmological constant. [13]

In this way, it has been found that BPS domain walls have many interesting properties using models with several *isolated* vacua. It is essential in these models that isolated vacua have different values of the superpotential, since their differences are related to the energy densities saturating the BPS bound. In many supersymmetric theories, however, the vacuum manifold consists of a *continuously degenerated*

moduli space. Since supersymmetric vacua are the extrema of the superpotential ($W' = 0$), the connected parts of the moduli space have the same values of the superpotential, and thus each connected part is mapped to a single *point* in the superpotential space. Hence we can expect the existence of domain walls in the models, with the moduli space composed of several disjoint parts, rather than isolated points, because these disjoint vacua (in the field space) are, in general, mapped to *different points* in the superpotential space. In short, the moduli spaces of disjoint supersymmetric vacua appear the same as the isolated vacua in the superpotential space.

In this work, we investigate BPS domain walls in $\mathcal{N} = 1$ four-dimensional supersymmetric field theories with continuous global symmetry. If the models have vacua with spontaneously broken global symmetry, there exists a flat direction along the broken symmetry, or the moduli space of the vacua. Domain walls in such theories can be expected to connect pairs of the vacua in the disjoint moduli spaces if the Homotopy group π_0 is nontrivial.¹ If a BPS domain wall connects such disjoint moduli spaces for broken global symmetry, the configuration itself breaks the symmetry. Hence there can be a family of BPS walls interpolating between two disjoint moduli spaces; the BPS bound for walls is given by the difference between the superpotential values corresponding to two vacua, and this never changes under the symmetry transformation. In fact, we show that applying a symmetry transformation to a BPS domain wall solution produces another solution of the BPS equation. Therefore we can expect *additional* moduli of BPS walls, in addition to the location of the wall's center.

There is another reason why we study the BPS walls in models with continuous global symmetry. It is known that when a supersymmetric model possesses global symmetry, the superpotential has a larger symmetry, or the complexification of the original global symmetry, owing to the holomorphy of the superpotential. The vacuum manifold has non-compact flat directions, corresponding to the imaginary parts

¹ Investigation of the Homotopy group gives the *necessary* conditions for the existence of domain walls, but there is not always a solution of the equation of motion, in particular the BPS equation for the BPS domain walls.

of the vacuum expectation values of the fields. The Nambu-Goldstone theorem for supersymmetric models has been proven. [18] From this, it is known that when a global symmetry is spontaneously broken in *supersymmetric* vacua, there appear NG supermultiplets as many as the number of the broken generators of the complexified group. Since the complexified group is the symmetry of the superpotential, *not* that of the whole model nor of the BPS equation, it is a highly nontrivial problem to determine whether there can exist BPS walls interpolating between two vacua along disjoint non-compact flat directions. We examine this problem by using two supersymmetric models with global $O(2)$ symmetry, consisting of two chiral superfields. Unlike the global $O(2)$ symmetry, $O(2)^{\mathbb{C}}$ transformations of a BPS wall solution are not solutions of the BPS equation. However, we show that there can exist moduli of BPS walls corresponding to the shift of vacua along the non-compact flat direction. This moduli is different from the imaginary part of the parameter of the $O(2)^{\mathbb{C}}$ transformation.

In sect. 2, we discuss the general properties of BPS domain walls in the model with continuous global symmetry. In sect. 3, we introduce our two models with $O(2)$ symmetry. We examine the existence of complex BPS walls interpolating between non-compact flat directions in both the models. In sect. 4, we reach conclusions for both models and discuss the features of BPS domain walls in general models with global symmetry. We also discuss a possible extension of the supersymmetric Nambu-Goldstone theorem.

2 BPS walls and continuous symmetry

We consider supersymmetric field theories with only chiral superfields, and the Kähler potential is assumed to be linear: $K = \phi^\dagger \phi$. The supersymmetric vacua are given as the extrema of the superpotential $W(\phi_k)$, given by

$$\frac{\partial W}{\partial \phi_k} = 0, \quad k = 1, \dots, K, \quad (2.1)$$

where the ϕ_k are the scalar components of the chiral superfields. It is known that, denoting two solutions of Eq. (2.1) by $\{\phi_k\}_I$ and $\{\phi_k\}_J$, and the corresponding values

of the superpotential by W_I and W_J , there exists the lower bound of the surface energy density, or tension, for walls connecting these two vacua expressed by

$$\mathcal{E} \equiv \frac{\text{Energy}}{\text{Area}} \geq 2|W_J - W_I|. \quad (2.2)$$

The BPS wall for which the equality in Eq. (2.2) holds satisfies the equation [8]

$$\partial_z \phi_k = e^{i\alpha} \frac{\partial W^*}{\partial \phi_k^*}, \quad (2.3)$$

where $\alpha = \arg(W_J - W_I)$. Here we have considered the wall depending on the coordinate z . Equation (2.3) is called the ‘‘BPS equation’’.

If the superpotential W is invariant under the global symmetry G ,

$$W(\phi) \rightarrow W(g\phi) = W(\phi), \quad \phi \xrightarrow{g} g\phi, \quad g \in G, \quad (2.4)$$

where ϕ belongs to unitary representation of G , Eq. (2.1) is also invariant under G :

$$\frac{\partial W(\phi)}{\partial \phi_i} \xrightarrow{g} g_{ij}^{-1T} \frac{\partial W(\phi)}{\partial \phi_j}. \quad (2.5)$$

Since the superpotential includes only chiral superfields, the invariant group G of the superpotential is enlarged to its complexification, $G^{\mathbf{C}}$. It is known that, in addition to the ordinary Nambu-Goldstone bosons corresponding to broken G symmetry, there appear so-called quasi-Nambu-Goldstone bosons corresponding to broken $G^{\mathbf{C}}$ symmetry.[18] With the fermions of their superpartner, they constitute massless chiral superfields. The vacuum manifold as a $G^{\mathbf{C}}$ -orbit is parameterized by these massless bosons, and the quasi-Nambu-Goldstone bosons just parameterize the non-compact flat directions.² Therefore, in the moduli space of its supersymmetric vacua, there exists a non-compact flat direction along the direction of the imaginary part of the scalar fields.

We can see that the BPS equation (2.3) is covariant under transformation of the global symmetry G , but it is not covariant under the transformation of $G^{\mathbf{C}}$, since the BPS equation includes both holomorphic and anti-holomorphic fields. Then, if we can find a solution of Eq. (2.3), configurations obtained through transformation

² It is known that, in the case of the F-term breaking, there must exist at least one quasi-Nambu-Goldstone boson. Then the vacuum manifold inevitably becomes non-compact.[19]

of this solution by elements of G are also solutions of the BPS equation. However, configurations obtained through transformations of a solution by elements of $G^{\mathbf{C}}$ are not generally solutions of the BPS equation. Therefore, if the model has more than two disjoint flat directions, it is a nontrivial problem to determine whether there exist BPS walls interpolating between them. We examine this problem in two supersymmetric models.

3 BPS walls in models with flat directions

3.1 Moduli spaces of our models with flat directions

In this paper, we consider the following two supersymmetric models with flat directions.³ First we consider a model with one flat direction. Its superpotential is

$$W(\phi) = \frac{1}{4}(\vec{\phi}^2 - a^2)^2, \quad \vec{\phi} = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}, \quad (3.1)$$

where ϕ^1 and ϕ^2 are chiral superfields composing the doublet of $O(2)$, $\vec{\phi}$, and a is a constant parameter. By a field redefinition, we can take this parameter a to be real and positive without loss of generality. This model has two disjoint vacua:

$$\begin{aligned} \text{Vac. I} \quad & \vec{\phi} = 0, & W &= \frac{a^4}{4}, \\ \text{Vac. II} \quad & \vec{\phi}^2 = a^2, & W &= 0. \end{aligned} \quad (3.2)$$

Let us note that the ϕ^i are the scalar components of chiral superfields here. (We denote the chiral superfields and their scalar components by the same letter.) Vac. I is $O(2)$ symmetric, but Vac. II spontaneously breaks $O(2)$ symmetry. The expectation value for Vac. II can be labeled as

$$\phi^1 = a \cos \theta, \quad \phi^2 = a \sin \theta. \quad (3.3)$$

³ The two models that we consider in this paper are not renormalizable. Therefore these models must be interpreted as effective theories.

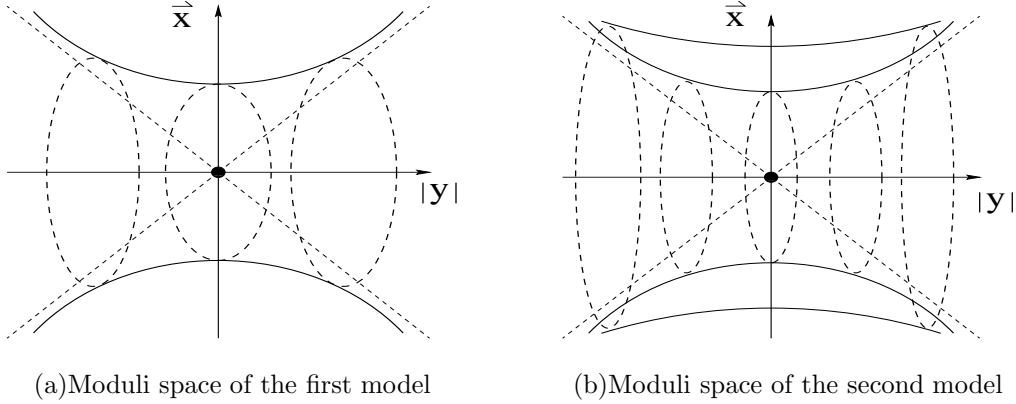


Figure 1: (a) The moduli space of the first model is composed of two disjoint parts: the origin and one hyperboloid. The hyperboloid has one compact direction, represented by the broken circles, and one non-compact direction, represented by the hyperbola. (b) The moduli space of the second model is composed of three disjoint parts: the origin and two hyperboloid with different sizes. In both (a) and (b), the horizontal axis is $|y| = \sqrt{y^2}$, the vertical axis is x^1 , and the axis orthogonal to them is x^2 . The smallest circle corresponds to the real moduli space of vacua, $\vec{y} = 0$, in both figures.

Now the fields ϕ^1 and ϕ^2 can take complex values, and we can regard θ as a *complex* parameter. Therefore the vacuum manifold of this model is enlarged to an $O(2)^{\mathbf{C}}$ -orbit: If we set $\vec{\phi} = \vec{x} + i\vec{y}$, the two disjoint vacua in the Eq. (3.2) become

$$\begin{aligned}
 \text{Vac. I} \quad & \vec{x} = \vec{y} = \vec{0}, \\
 \text{Vac. II} \quad & \vec{x}^2 - \vec{y}^2 = a^2, \text{ and } \vec{x} \cdot \vec{y} = 0.
 \end{aligned} \tag{3.4}$$

Hence Vac. II can be rewritten as a two-dimensional surface in the three-dimensional linear space $(x^1, x^2, |y|)$, where $|y| = \sqrt{\vec{y}^2}$ (see Fig. 1). Vac. II breaks this $O(2)^{\mathbf{C}}$ symmetry spontaneously. We consider the BPS wall connecting $O(2)^{\mathbf{C}}$ symmetric and $O(2)^{\mathbf{C}}$ broken vacua, and show that no BPS wall can connect the *complex vacuum* - the vacuum with a complex value of the fields shifting along the flat direction in this model (see Fig. 1).

Next we consider the model with two flat directions. Its superpotential is

$$W(\phi) = \frac{1}{6} \vec{\phi}^2 (\vec{\phi}^2 - a^2)^2, \tag{3.5}$$

where $\vec{\phi}$ is an $O(2)$ doublet composed of the chiral superfields ϕ^1 and ϕ^2 , and the parameter a is assumed to be a positive real constant for simplicity. This model has three disjoint vacua:

$$\begin{aligned} \text{Vac. I} \quad & \vec{\phi} = 0, & W = 0, \\ \text{Vac. II} \quad & \vec{\phi}^2 = \frac{a^2}{3}, & W = \frac{2}{81}a^6, \\ \text{Vac. III} \quad & \vec{\phi}^2 = a^2, & W = 0. \end{aligned} \tag{3.6}$$

Setting $\vec{\phi} = \vec{x} + i\vec{y}$, as in the previous model, Vac. II and Vac. III can be rewritten as two hyperboloids with different sizes and Vac. I as the origin in the space $(x^1, x^2, |y|)$ (see Fig. 1). We see that Vac. I is $O(2)^{\mathbb{C}}$ symmetric, but Vac. II and Vac. III break $O(2)^{\mathbb{C}}$ symmetry spontaneously. We consider the two kinds of BPS walls, connecting Vac. I and Vac. II, and connecting Vac. II and Vac. III. Then we show that the BPS walls can connect the complex vacua of Vac. II and Vac. III, but cannot connect Vac. I and complex vacua of Vac. II.

3.2 BPS walls in model I

Here, we construct BPS saturated walls in the model with one flat direction (Model I). The BPS equation (2.3) for this wall is

$$\frac{\partial \phi^i}{\partial z} = \phi^{*i}(\vec{\phi}^{*2} - a^2). \tag{3.7}$$

First we show that there is no complex solution of this BPS equation. When we map the field space to the superpotential space, two disjoint vacua are mapped to two points. It is known that the configuration of the BPS wall can be mapped to a line segment connecting these two points in the superpotential space. [20] Now, the difference between the values of the superpotentials for the two vacua, $\Delta W = a^4/4$, is real. This means that the configuration of the BPS wall in the superpotential space is also real. If we set $\vec{\phi} = \vec{x} + i\vec{y}$, the imaginary part of the superpotential is $\Im W = 4(\vec{x} \cdot \vec{y})(\vec{x}^2 - \vec{y}^2 - a^2)$, so we find that BPS solution must satisfy the constraint $\vec{x} \cdot \vec{y} = 0$. Using this constraint, the BPS equation of Eq. (3.7) can be rewritten as

$$\frac{d}{dz}(\vec{x} + i\vec{y}) = (\vec{x} - i\vec{y})(\vec{x}^2 - \vec{y}^2 - a^2). \tag{3.8}$$

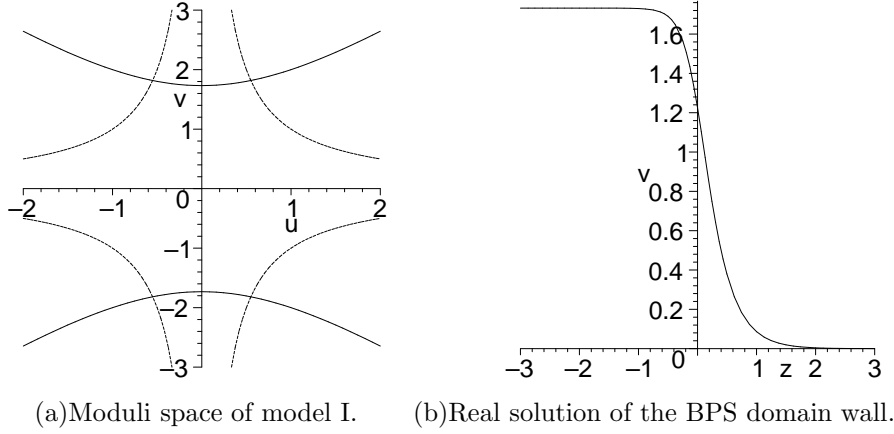


Figure 2: (a) The moduli space of model I in the (u, v) -plane is represented by the solid curves, and the broken curves correspond to $uv = \pm\sqrt{c}$ (for $c = 1$). (We set $a^2 = 3$ in all of the figures in this paper.) (b) The real solution, Eq. (3.12), connecting the origin and the nearest points in the hyperbola is plotted.

From this equation we can derive the following equations:

$$\frac{d}{dz} \left(\frac{x^2}{x^1} \right) = \frac{d}{dz} \left(\frac{y^1}{y^2} \right) = 0, \quad \frac{d}{dz} (x^i y^j) = 0, \quad \text{for } i, j = 1, 2. \quad (3.9)$$

The first of these two equations implies that the $O(2)$ rotation parameter θ is constant for the BPS wall. Combining these with the constraint $\vec{x} \cdot \vec{y} = 0$, we can parameterize the BPS wall as

$$\vec{\phi}(z) = v(z) \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} + iu(z) \begin{pmatrix} \cos(\theta + \pi/2) \\ \sin(\theta + \pi/2) \end{pmatrix}. \quad (3.10)$$

In Fig. 2 (a), we plot the moduli space of this model in the (u, v) -plane. Substituting Eq. (3.10) into the second equation of Eq. (3.9), we can immediately find

$$\frac{d(uv)}{dz} = 0, \quad uv = \text{const} \equiv \sqrt{c}, \quad (3.11)$$

where c is a real integral constant. From Fig. 2(a) we find that there is no *complex* BPS solution connecting Vac. I and vacua along the flat direction of Vac. II: In order for a BPS wall to reach Vac. I, we need to set $uv = \sqrt{c} = 0$, and this is reduced to a real solution $[u(z) = 0]$ for the boundary condition of Vac. II on the other side.

Hence we consider this solution of Eq. (3.8). This solution can be found as

$$v = \phi_W \equiv a \sqrt{\frac{1}{1 + \exp[2a^2(z - z_0)]}}, \quad u = 0, \quad (3.12)$$

where z_0 is an integral constant, representing the position of the center of the domain wall. We plot this real solution in Fig. 2(b). Using an $O(2)$ transformation, the general real solutions can be written as

$$\phi^1 = \phi_W \cos \theta, \quad \phi^2 = \phi_W \sin \theta, \quad (3.13)$$

where θ is a *real* parameter. The wall separates the two vacua in the broken phase and the unbroken phase. The wall interpolating between the broken and unbroken phase of the *discrete* symmetry Z_2 is discussed in Ref. [21].

3.3 BPS wall in the model II

In this section, we construct BPS walls in the model with two flat directions (model II). This model has three disjoint vacua as in the case of Eq. (3.6). The difference between the values of the superpotentials for each pair of the three vacua is real, as in the previous model. There exists no BPS wall connecting Vac. I and Vac. III, because the two values of the superpotential corresponding to these two vacua are the same, and the BPS bound (2.2) becomes zero. For this reason we consider two kinds of walls: walls interpolating between Vac. II and Vac. III (“outer walls”), and walls interpolating between Vac. I and Vac. II (“inner walls”). The BPS equations (2.3) for these walls are

$$\frac{\partial \phi^i}{\partial z} = \phi^{*i} \left(\vec{\phi}^{*2} - \frac{a^2}{3} \right) (\vec{\phi}^{*2} - a^2), \quad (3.14)$$

where the boundary conditions are $\vec{\phi}(-\infty) = a^2$ [$\vec{\phi}(-\infty) = 0$] and $\vec{\phi}(\infty) = a^2/3$ for the outer (inner) walls.

The map of the BPS walls into the superpotential space must be real, as in the previous model: If we set $\vec{\phi} = \vec{x} + i\vec{y}$, the imaginary part of the superpotential in this model becomes

$$\Im W = \frac{1}{3}(\vec{x} \cdot \vec{y})[3(\vec{x}^2 - \vec{y}^2 - a^2)(\vec{x}^2 - \vec{y}^2 - a^2/3) - 4(\vec{x} \cdot \vec{y})^2]. \quad (3.15)$$

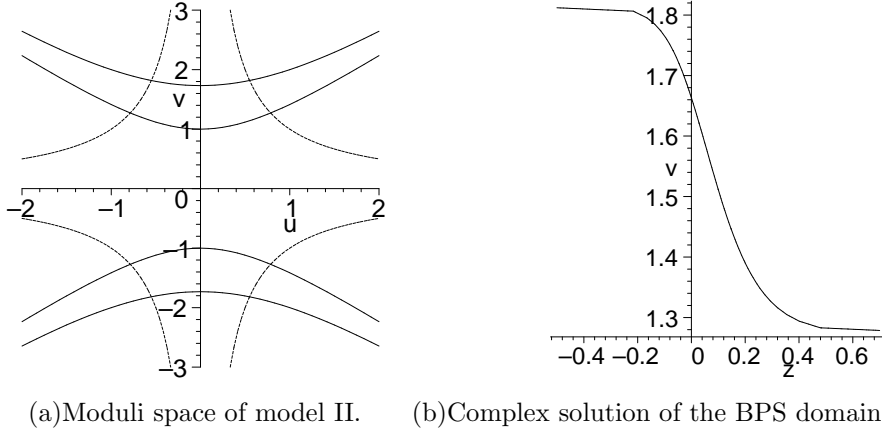


Figure 3: (a) The moduli space of model II in the (u, v) -plane is represented by the solid curves, and the broken curves correspond to $uv = \pm\sqrt{c}$ (for $c = 1$). The value of v for the complex solution, Eq. (3.20), connecting between the two hyperbolas along $uv = \sqrt{c}$ ($c = 1$) is plotted in (b).

Thus $\vec{x} \cdot \vec{y} = 0$ is a sufficient condition.⁴ With this condition, Eq. (3.9) is again valid, and we can set $\vec{\phi}$ as in Eq. (3.10). Hence we can set $\theta = 0$ in Eq. (3.10) by using the $O(2)$ transformation, without loss of generality, yielding $\vec{\phi} = \begin{pmatrix} v \\ iu \end{pmatrix}$, where v and u are real scalar fields. In Fig. 3, we illustrate the moduli space of this model in the (u, v) -plane. Equation (3.14) becomes

$$\begin{aligned} \frac{dv}{dz} &= v \left(v^2 - u^2 - \frac{a^2}{3} \right) (v^2 - u^2 - a^2), \\ \frac{du}{dz} &= -u \left(v^2 - u^2 - \frac{a^2}{3} \right) (v^2 - u^2 - a^2). \end{aligned} \quad (3.16)$$

We can then find

$$\frac{d(uv)}{dz} = 0. \quad (3.17)$$

Hence, we can set $uv = \text{const} = \sqrt{c}$. We find, from Fig. 3 (a), that there can exist a complex BPS wall solution connecting Vac. II and Vac. III, but no complex BPS

⁴ We can show that this is also a necessary condition using the continuity of the solution.

wall can connect Vac. I and Vac. II, for the same reason as in model I. The first equation in Eq. (3.16) becomes

$$\frac{dv^2}{dz} = -2\frac{1}{v^2} \left((v^2)^2 - \frac{a^2}{3}v^2 - c \right) ((v^2)^2 - a^2v^2 - c). \quad (3.18)$$

This can be integrated to give

$$e^{-\frac{4a^2}{3}(z-z_0)} = \left| \frac{v^2 - \frac{1}{2}(\frac{a^2}{3} + \sqrt{\frac{a^4}{9} + 4c})}{v^2 - \frac{1}{2}(\frac{a^2}{3} - \sqrt{\frac{a^4}{9} + 4c})} \right|^{\frac{1}{\sqrt{\frac{a^4}{9} + 4c}}} \left| \frac{v^2 - \frac{1}{2}(a^2 - \sqrt{a^4 + 4c})}{v^2 - \frac{1}{2}(a^2 + \sqrt{a^4 + 4c})} \right|^{\frac{1}{\sqrt{a^4 + 4c}}}, \quad (3.19)$$

where z_0 is the center of the wall. For the complex solution interpolating between Vac. II and Vac. III, (3.19) can be rewritten as

$$e^{-\frac{4a^2}{3}(z-z_0)} = \left[\frac{v^2 - \frac{1}{2}(\frac{a^2}{3} + \sqrt{\frac{a^4}{9} + 4c})}{v^2 - \frac{1}{2}(\frac{a^2}{3} - \sqrt{\frac{a^4}{9} + 4c})} \right]^{\frac{1}{\sqrt{\frac{a^4}{9} + 4c}}} \left[\frac{v^2 - \frac{1}{2}(a^2 - \sqrt{a^4 + 4c})}{\frac{1}{2}(a^2 + \sqrt{a^4 + 4c}) - v^2} \right]^{\frac{1}{\sqrt{a^4 + 4c}}} \quad (3.20)$$

Since we cannot obtain an explicit solution $v(z)$ of this equation, we plot $v(z)$ in the Fig. 3(b) as an implicit solution of a complex BPS wall.

We must note that the complex solution of $uv = \sqrt{c}$ is not the $O(2)^{\mathbb{C}}$ transformation of the solution of $uv = 0$. Let us consider a vacuum transformed by a $O(2)^{\mathbb{C}}$ parameter from a real expectation value in Vac. II. The complex BPS wall solution connects this Vac. II to the Vac. III transformed by a different $O(2)^{\mathbb{C}}$ parameter from the corresponding real expectation value in Vac. III. Therefore the $O(2)^{\mathbb{C}}$ transformation of a BPS solution does not become a solution of the BPS equation; the parameter c which labels the imaginary direction is not associated with the $O(2)^{\mathbb{C}}$ symmetry.

We can find an explicit solution for real BPS walls. For the real solution, the integrated BPS equation can be obtained by setting $c = 0$ in Eq. (3.19). We have

$$X \stackrel{\text{def}}{=} \exp \left[\frac{4a^4}{3}(z - z_0) \right] = \frac{|v^2 - a^2|v^4}{|v^2 - \frac{a^2}{3}|^3} = \frac{|\Phi - a^2|\Phi^2}{|\Phi - \frac{a^2}{3}|^3}, \quad (3.21)$$

where we have defined $\Phi \stackrel{\text{def}}{=} v^2 = (\phi^1)^2$.

We solve this equation in the outer region, $\frac{a^2}{3} \leq (\phi^1)^2 \leq a^2$, and the inner region, $0 \leq (\phi^1)^2 \leq \frac{a^2}{3}$, separately.

In the case of the outer solutions, $\frac{a^2}{3} \leq (\phi^1)^2 \leq a^2$, Eq. (3.21) can be rewritten as the third order equation

$$(X+1)\Phi^3 - a^2(X+1)\Phi^2 + \frac{a^4}{3}X\Phi - \frac{a^6}{27}X = 0. \quad (3.22)$$

Thus the third order equation can be solved to yield

$$(\phi^1)^2 = \frac{a^2}{3} \left[1 + \left(\frac{1}{1+X} + \sqrt{\frac{X}{(X+1)^3}} \right)^{\frac{1}{3}} + \left(\frac{1}{1+X} - \sqrt{\frac{X}{(X+1)^3}} \right)^{\frac{1}{3}} \right] \quad (3.23)$$

for a real solution. (The two other solutions are complex and thus inappropriate.)

In the case of the inner solutions, $0 \leq (\phi^1)^2 \leq \frac{a^2}{3}$, Eq. (3.21) can be rewritten as

$$(X-1)\Phi^3 - a^2(X-1)\Phi^2 + \frac{a^4}{3}X\Phi - \frac{a^6}{27}X = 0. \quad (3.24)$$

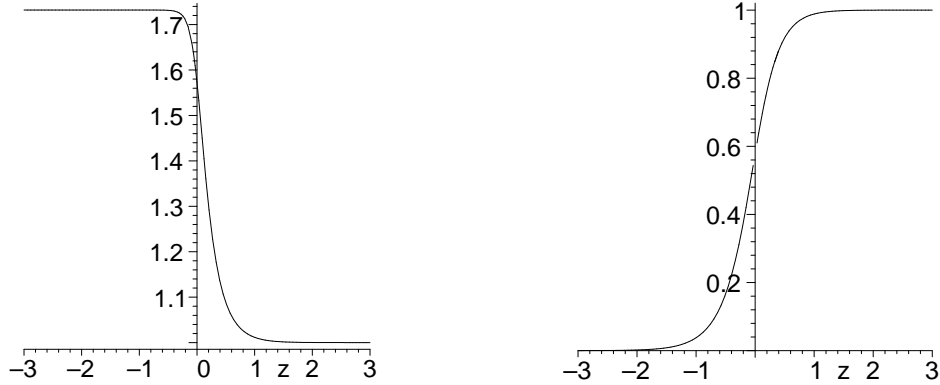
In this case, we must solve this equation for each case $X = 1$ and $X \neq 1$ separately. When $X = 1$, the solution of this equation is $(\phi^1)^2 = a^2/9$, and this corresponds to the expectation value at the center of the wall ($z = z_0$). When $X \neq 1$ ($z \neq z_0$), there are three candidates for the solution of the outer wall:

$$\begin{aligned} (\phi^1)^2 = \frac{a^2}{3} & \left[1 + \left(\frac{1}{1-X} + \sqrt{\frac{X}{(X-1)^3}} \right)^{\frac{1}{3}} \begin{pmatrix} 1 \\ e^{\frac{2\pi}{3}i} \\ e^{-\frac{2\pi}{3}i} \end{pmatrix} \right. \\ & \left. + \left(\frac{1}{1-X} - \sqrt{\frac{X}{(X-1)^3}} \right)^{\frac{1}{3}} \begin{pmatrix} 1 \\ e^{-\frac{2\pi}{3}i} \\ e^{\frac{2\pi}{3}i} \end{pmatrix} \right]. \end{aligned} \quad (3.25)$$

These solutions are not real and positive, so we must choose the correct one for the regions $z < z_0$ ($X < 1$) and $z > z_0$ ($X > 1$). In the region $z > z_0$, the first solution is appropriate for the real solution. In the region of $z < z_0$, the third solution is appropriate. (In the latter case, the first solution cannot satisfy the correct boundary conditions, $(\phi^1)^2(-\infty) = 0$, and the second solution tends to infinity in the limit $z \rightarrow z_0$.) In summary, we obtain the inner wall solution by using the third solution in the left ($z < z_0$) and the first solution in the right ($z > z_0$):

$$(\phi^1)^2 = \begin{cases} \frac{a^2}{3} \left[1 + \left(\frac{1}{1-X} + \sqrt{\frac{X}{(X-1)^3}} \right)^{\frac{1}{3}} - \left(-\frac{1}{1-X} + \sqrt{\frac{X}{(X-1)^3}} \right)^{\frac{1}{3}} \right] & (z > z_0) \\ \frac{a^2}{3} \left[1 + \left(\frac{1}{1-X} + \sqrt{\frac{X}{(X-1)^3}} \right)^{\frac{1}{3}} e^{-\frac{2\pi}{3}i} + \left(\frac{1}{1-X} - \sqrt{\frac{X}{(X-1)^3}} \right)^{\frac{1}{3}} e^{\frac{2\pi}{3}i} \right] & (z < z_0) \end{cases} \quad (3.26)$$

The profiles of the outer and inner wall solutions are plotted in Fig. 4.



(a) Outer solution of the BPS domain wall. (b) Inner solution of the BPS domain wall.

Figure 4: The two kinds of real solutions, the outer solution, Eq. (3.23), and the inner solution, Eq. (3.26), are plotted in (a) and (b), respectively.

4 Conclusions and discussion

We considered BPS domain walls in models with continuously degenerate moduli spaces. We discussed only two $O(2)$ symmetric models explicitly, but many results can be straightforwardly generalized to other models with a global symmetry G . When a model has a continuous symmetry, $O(2)$ in our models, the BPS equation of the wall becomes covariant under this symmetry, so the BPS wall also has this symmetry. If we can find a BPS solution, configurations obtained through transformations of this solution by elements of G are also BPS solutions, so they constitute a family of BPS walls. Although the boundary conditions change under these transformations, the tensions of the walls never change.

In supersymmetric field theories, the symmetry G of the superpotential is enlarged to its complexification, $G^{\mathbb{C}}$, due to the holomorphy of the superpotential. Therefore the vacuum manifold includes non-compact flat directions corresponding to the directions of imaginary parts of the vacuum expectation values. As the BPS equation is not covariant under $G^{\mathbb{C}}$, it is a highly nontrivial problem to determine whether there can exist *complex* BPS walls interpolating between two disjoint non-compact flat directions. To examine this problem, we considered two models with

flat directions. We found that there is no complex BPS wall in the first model, while there can exist a family of *complex* BPS walls in the second model. We have learned from the examination of these two models that we must consider *complex* configurations for BPS walls in models with continuous symmetry. This is an important lesson, since only real configurations of BPS walls have been considered in the literature.

We have not yet found a criterion to determine whether or not a complex BPS wall exists in general models. Let us now examine general structures for the existence of complex BPS walls by counting the number of degrees of freedom in these two models. Since the BPS equation is a first order differential equation, it can be expected that the general solution has the same number of integral constants as the number of BPS equations, *unless we enforce the boundary conditions*. Since we considered supersymmetric models with two chiral superfields, there are four BPS equations corresponding to the four real scalar degrees of freedom. However, we have been able to eliminate one degree of freedom, since it must be the case of that any BPS solution maps to a straight line in the superpotential space. We thus can expect that BPS solutions can *maximally* include three free parameters as the integral constants. In fact, three parameters, z_0 , θ and c , have appeared as the integral constants in the BPS solutions in the second model. However, the third parameter c is not contained in the BPS wall solution of the first model: It was eliminated by the boundary condition.

We can interpret the parameters z_0 and θ (and c), labeling the solutions of the BPS walls, as the “moduli” of the BPS wall solutions, since the tension of the wall does not change when we continuously vary the values of these parameters. The configurations obtained under such variations are all solutions of a BPS equation, and their tension realize the *same* BPS bound. These parameters, however, have slightly different meanings: since z_0 represents the location of the center of the wall, we can vary this parameter *without* changing the boundary conditions. Contrastingly, we *cannot* vary θ and c without changing the boundary condition.

Next we discuss the nature of these parameters in terms of symmetry. Two of the three parameters represent the Nambu-Goldstone modes corresponding to

the symmetries broken by the existence of the wall configuration; z_0 corresponds to translation along the z -axis in the spacetime, and θ to the continuous internal symmetry, $O(2)$. Therefore the BPS wall solution apparently contains these free parameters. The parameter c can be considered to represent the deformation of the BPS wall along the non-compact flat direction, which originates from the complexified symmetry of the superpotential. This is, however, *not* the symmetry of the whole model (the Kähler potential is invariant under G but not $G^{\mathbb{C}}$), and therefore the additional parameter c does not directly correspond to the complex symmetry. This is why the BPS wall solutions do not always contain c . Concerning this fact, we must comment on the similarity with the results in Ref. [9]. As discussed above, the parameter c in our model is the additional integral constant, which depends on the details of the model. This quantity is similar to the additional integrals of motion in Ref. [9],⁵ in the sense that in both models the additional constants do not correspond directly to the symmetry of the theory. However, we must emphasize that these quantities have essentially different origins: The additional integrals of motion in Ref. [9] represent the spatial distance (in the *spacetime*) of two separated BPS walls, while the quantity c in our model controls the shift of the BPS walls along the flat direction in the *internal* space.

Let us discuss an interesting problem regarding the Nambu-Goldstone theorem suggested by our models. The moduli space of supersymmetric vacua is parameterized by the NG and the quasi-NG bosons associated with the spontaneously broken $G^{\mathbb{C}}$ symmetry of the superpotential, and with their superpartners they constitute massless NG chiral multiplets as described by the supersymmetric extension of the Nambu-Goldstone theorem. [18] However, the configuration of the BPS domain wall spontaneously breaks half of the supersymmetry (and the translational symmetry along the z -axis). Therefore, in the entire four-dimensional spacetime, $\mathcal{N} = 1$ massless NG supermultiplets are justified only at infinite distance from the wall. The supersymmetric Nambu-Goldstone theorem must be deformed around the wall. This fact may be a reason why the complex parameter of the $O(2)^{\mathbb{C}}$ transforma-

⁵ Similar additional constants are discussed in the context of non-supersymmetric models in Ref. [22].

tion does not appear as the moduli of BPS wall solutions. It would be interesting to examine the extension of the Nambu-Goldstone theorem to the case of a BPS wall background, or the case in which half of the supersymmetry is spontaneously broken.

Before ending this conclusion, we point out some interesting features of our models. We found that there can exist BPS walls connecting $O(2)$ symmetric and $O(2)$ broken vacua. (For conventional BPS walls, the broken symmetry is usually discrete, and vacua separated by the wall are *both* in the broken phase.) Mass spectra are different on opposite sides of the walls in our models: We can expect massless (quasi-)NG bosons and their superpartners only in the broken phase. It is a future problem to examine the wave functions of these massless modes in order to determine this difference.

Our second model has three disjoint vacua, and the maps from two of them to the superpotential space coincide accidentally. By modifying the model slightly, we can construct a model with three disjoint vacua mapped to three distinct points in the superpotential space. Hence our examinations can be extended to the case of the BPS domain wall junction.

We expect that the new types of BPS domains wall found in this paper will play an important role in the further understanding of non-perturbative aspects of supersymmetric quantum field theories.

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